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SUMMARY

A study is made of the effect of the flexural stiffness of the face sheets on the buckling of elastic curved plates and cylindrical shells of sandwich construction subjected to axial compression. The study shows that when the core is very weak in shear, the flexural stiffness of the face sheets can have an important effect on the buckling load. Simple formulas are developed which give good approximations to the buckling load for most practical ranges of the parameters where the effect of face-sheet stiffness is important. Results of these formulas are compared with exact results for an infinitely long curved plate over a large range of sandwich-shell parameters. The results are based on a linear buckling theory for sandwich shells of the Donnell type (NACA Report 479), which takes into account the asymmetry of the sandwich faces.

INTRODUCTION

Over the past few years a number of studies have been made of the buckling characteristics of curved plates and cylindrical shells of sandwich construction subjected to axial compression. In most of these studies the contribution of the flexural stiffness of the face sheets to the buckling characteristics has been neglected. Although the assumption that face-sheet contribution should be neglected is reasonable in certain ranges of the significant parameters, it may not be reasonable in the range where large shearing deformations occur in the core at buckling. Buckling in this latter range of parameters results in small axial buckle wavelengths; consequently, the faces undergo large bending deformations. The strain energy associated with this bending may be significant, and the effects of face-sheet stiffness should be considered in determining the buckling load of the shell.

The purpose of this paper is to investigate the contribution of the flexural stiffness of the face sheets to the buckling load of a curved plate or cylindrical shell of sandwich construction in axial compression. The investigation shows that a simple expression for the buckling load including this face-sheet contribution can be developed which holds for most practical ranges of the parameters where the face-sheet contribution is important. The present study is an extension of the work carried out by Stein and Mayers (ref. 1) where the face-sheet stiffness was neglected.

SYMBOLS

a,b coordinates of edges of shell in x- and y-directions, respectively

B extensional stiffness of face sheet, $\dfrac{Et}{1 - \mu^2}$

c thickness of core

D flexural stiffness of face sheet, $\dfrac{Et^3}{12(1 - \mu^2)}$

$d = \dfrac{cB_1B_2}{G_c(B_1 + B_2)}$

$\bar{d} = \dfrac{h^2B_1B_2}{B_1 + B_2}$

E Young's modulus for face sheet

F Airy stress function

G_c shear modulus for core

h distance between middle surfaces of face sheets

k_a buckling-load coefficient for cylinder, $\dfrac{N^*a^2(B_1 + B_2)}{\pi^2h^2B_1B_2}$

k_b buckling-load coefficient for curved plate, $\dfrac{N^*b^2(B_1 + B_2)}{\pi^2h^2B_1B_2}$

m,n integers

M_x,M_y sum of bending moments in face sheets

N_{cr} critical axial stress resultant

N_x, N_y, N_{xy}	buckling increments in normal and shear stress resultants
N^*	externally applied axial stress resultant
N_x^*, N_y^*, N_{xy}^*	externally applied normal and shear stress resultants
r	radius of curvature of shell
S	face-sheet-stiffness parameter, $\frac{(B_1 + B_2)(D_1 + D_2)}{h^2 B_1 B_2}$
t	thickness of face sheet
u,v,w	buckling displacements of shell in x-, y-, and z-directions, respectively (fig. 1)
w_o	constant (eq. (6))
x,y,z	coordinates of middle surface of shell (fig. 1)
Z_a	curvature parameter for cylinder $\left(Z_a^2 = \frac{a^4(B_1 + B_2)^2(1 - \mu^2)}{r^2 h^2 B_1 B_2}\right)$
Z_b	curvature parameter for plate $\left(Z_b^2 = \frac{b^4(B_1 + B_2)^2(1 - \mu^2)}{r^2 h^2 B_1 B_2}\right)$
α	buckling rotation in x-direction, $\frac{1}{h}(u_1 - u_2)$
β	buckling rotation in y-direction, $\frac{1}{h}(v_1 - v_2)$
λ	length-width ratio, $\frac{a}{b}$
μ	Poisson's ratio
φ	coupled rotation variable, $\alpha_{,x} + \beta_{,y}$
ψ_a	sandwich-core parameter for cylinder, $\frac{\pi^2 c B_1 B_2}{a^2 G_c (B_1 + B_2)}$

ψ_b	sandwich-core parameter for curved plate, $\dfrac{\pi^2 c B_1 B_2}{b^2 G_c \left(B_1 + B_2\right)}$
∇^2	two-dimensional Laplacian operator

Subscripts:

i	integer 1 or 2
1,2	refer to upper and lower face sheets, respectively
x,y	after commas, indicate partial differentiation with respect to axial and circumferential coordinates, respectively

GENERAL BUCKLING EQUATIONS

Assumptions in the Sandwich Theory

The buckling equations to be solved were obtained from the nonlinear equations originally developed in references 2 and 3. The variables in the nonlinear equations were separated into prebuckling and buckling effects in a manner similar to that in reference 4. In the present study boundary conditions on the prebuckling state have been relaxed so that the prebuckling displacements are either constant or linear.

The concept of a sandwich is retained, in that the core undergoes only transverse shear deformations so that a line through the undeformed core remains straight when the core is deformed but does not necessarily remain perpendicular to the neutral surface of the shell. It is assumed that the total thickness of the shell element is small compared with the radius of curvature. The face sheets are assumed to be elastic, isotropic, and homogeneous and to follow classical shell theory - that is, to satisfy the Kirchhoff-Love condition. The core is assumed to be elastic, isotropic, and homogeneous, to carry no inplane loads, and to have no deformation in the direction normal to the neutral surface of the shell. Poisson's ratio is taken to be the same for the two face sheets and the core; however, the thicknesses and Young's modulus may be different.

Governing Differential Equations

If the above assumptions are employed, the three differential equations governing the buckling of a cylindrical shell of a nonsymmetrical sandwich section (fig. 1) are as follows:

$$\nabla^4 F = -\left(B_1 + B_2\right)\left(1 - \mu^2\right)\frac{w_{,xx}}{r} \tag{1a}$$

$$\left(1 - d\nabla^2\right)\varphi = \nabla^2 w \tag{1b}$$

$$\left(D_1 + D_2\right)\nabla^4 w - N_x^* w_{,xx} - 2N_{xy}^* w_{,xy} - N_y^* w_{,yy} - \frac{1}{r} F_{,xx} + \bar{d}\nabla^2\varphi = 0 \tag{1c}$$

where

$$\left.\begin{aligned} &\varphi = \alpha_{,x} + \beta_{,y} \qquad d = \frac{cB_1B_2}{G_c\left(B_1 + B_2\right)} \qquad \bar{d} = \frac{h^2B_1B_2}{B_1 + B_2} \\ &\alpha = \frac{1}{h}\left(u_1 - u_2\right) \qquad \beta = \frac{1}{h}\left(v_1 - v_2\right) \\ &D_i = \frac{E_i t_i^3}{12\left(1 - \mu^2\right)} \qquad B_i = \frac{E_i t_i}{\left(1 - \mu^2\right)} \qquad (i = 1 \text{ or } 2) \end{aligned}\right\} \tag{2}$$

The quantities u_i, v_i, and w $(w_i = w)$ are the middle-surface incremental buckling displacements of the ith (upper or lower) face sheet of the sandwich in the longitudinal (x), circumferential (y), and inward radial (z) directions, respectively. Young's modulus for the face sheets is denoted by E_i, Poisson's ratio by μ, and the shear modulus of the core by G_c. The thickness of the ith face sheet is given by t_i, the thickness of the core by c, the distance between face-sheet centers by h, and the radius of curvature of the shell by r. The subscripts x and y after a comma indicate differentiation with respect to the axial and circumferential coordinates, respectively. The Laplacian operator is given by ∇^2, and F is the Airy stress function such that the buckling increments in the normal stress resultants N_x and N_y and in the shear stress resultant N_{xy} are given by

$$\left.\begin{aligned} N_x &= F_{,yy} \\ N_y &= F_{,xx} \\ N_{xy} &= -F_{,xy} \end{aligned}\right\} \tag{3}$$

The inplane forces which exist in the shell just prior to buckling are denoted by N_x^*, N_y^* (assumed to be positive in tension), and N_{xy}^*.

The buckling behavior of the sandwich shell can be determined from a solution to equations (1) with the appropriate boundary conditions in terms of the three unknowns, w, F, and φ. The usual shell theory, which includes the effect of shearing deformations, requires that five conditions be satisfied on the boundary. For equations (1), however, six conditions are necessary. The extra condition results from both including the flexural stiffnesses of the face sheets in resisting the total moment and retaining the Kirchhoff-Love assumption for the individual faces. Thus, although in classical shell theory only one boundary condition is required for the total moment normal to an edge, the present study requires two separate conditions, which separate the total moment into a moment in the faces and a moment couple resulting from inplane forces in the faces.

In equation (1b) the variable φ results from coupling, through differentiation and addition, the two equations which define α and β, the rotations in the longitudinal and circumferential directions, respectively. (See ref. 2.) A complete set of boundary conditions for equations (1) should therefore be specified in terms of α and β, not just φ. In the present study, however, it is convenient to use equation (1b) because the boundary conditions are such that φ, rather than α and β, is a natural variable.

BUCKLING OF A CURVED PLATE

Plate of Finite Length

The problem of interest is to determine the buckling load of a rectangular, simply supported, cylindrical plate subjected to a normal compressive force N^* parallel to its directrix along the edges $x = 0$ and $x = a$ (fig. 2). For this loading condition, $N_x^* = -N^*$ and $N_{xy}^* = N_y^* = 0$.

The six boundary conditions which govern buckling and which are defined herein as simply supported are given as follows along $x = 0$ and $x = a$:

(1) Displacement normal to the surface of the plate vanishes; that is,

$$w = 0 \tag{4a}$$

(2) Moment couple normal to the edge due to differential inplane forces in the face sheets vanishes; that is,

$$\alpha,_x + \mu\beta,_y = 0 \tag{4b}$$

(3) The sum of the moment in each of the individual face sheets vanishes; that is,

$$M_x = -\left(D_1 + D_2\right)\left(w,_{xx} + \mu w,_{yy}\right) = 0 \tag{4c}$$

(4,5) Displacements parallel to each edge are prevented; that is,

$$v = \frac{B_1 v_1 + B_2 v_2}{B_1 + B_2} = 0 \tag{4d}$$

$$\beta = 0 \tag{4e}$$

(6) Motion normal to each edge in the plane of the sheet occurs freely; that is,

$$F_{,yy} = 0 \tag{4f}$$

A comparable set of conditions along $y = 0$ and $y = b$ are:

$$w = 0 \tag{5a}$$

$$\beta_{,y} + \mu\alpha_{,x} = 0 \tag{5b}$$

$$M_y = -\left(D_1 + D_2\right)\left(w_{,yy} + \mu w_{,xx}\right) = 0 \tag{5c}$$

$$u = \frac{B_1 u_1 + B_2 u_2}{B_1 + B_2} = 0 \tag{5d}$$

$$\alpha = 0 \tag{5e}$$

$$F_{,xx} = 0 \tag{5f}$$

It should be noted that equations (4b), (4e), (5b), and (5e) imply that φ vanishes on the boundary. The quantities M_x and M_y are the sum of the moments in the two face sheets and u and v are the neutral surface displacements in the x- and y-directions, respectively.

The solution to equations (1) is

$$w = w_o \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{6}$$

$$F = \left(B_1 + B_2\right)\left(1 - \mu^2\right)\frac{w_o}{r} \frac{m^2}{\pi^2 a^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{7}$$

$$\varphi = -\frac{w_o \pi^2\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}{1 + \pi^2 d\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{8}$$

where w_o is a constant and m and n are integers. Equations (6), (7), and (8) satisfy explicitly all the applicable boundary conditions (eqs. (4) and (5)) except those given by equations (4d) and (5d). It can be seen that the conditions given by equations (4d) and (5d) are also satisfied by noting that u and v are related to w and F by (ref. 2)

$$\left.\begin{aligned} u_{,x} &= \frac{F_{,yy} - \mu F_{,xx}}{\left(1 - \mu^2\right)\left(B_1 + B_2\right)} \\ v_{,y} &= \frac{F_{,xx} - \mu F_{,yy}}{\left(1 - \mu^2\right)\left(B_1 + B_2\right)} + \frac{w}{r} \\ u_{,y} + v_{,x} &= \frac{-2F_{,xy}}{\left(1 - \mu\right)\left(B_1 + B_2\right)} \end{aligned}\right\} \tag{9}$$

Substituting equations (6), (7), and (8) into equation (1c) yields

$$\left\{-N^*\pi^2 \frac{m^2}{a^2} + \left(D_1 + D_2\right)\pi^4 \frac{\left(m^2 + \lambda^2 n^2\right)}{a^4} + \frac{\left(B_1 + B_2\right)\left(1 - \mu^2\right)m^4}{r^2\left(m^2 + \lambda^2\pi^2\right)^2}\right.$$

$$\left. + \frac{\bar{d}\pi^4\left(m^2 + \lambda^2 n^2\right)^2}{a^4\left[1 + \frac{\pi^2 d}{a^2}\left(m^2 + \lambda^2 n^2\right)\right]}\right\} w_o = 0 \tag{10}$$

The trivial solution corresponding to the unbuckled state occurs for $w_o = 0$, and the buckling load is obtained by setting the braced term equal to zero. The equation for the buckling-load coefficient is therefore

$$k_b = \frac{S\left(1 + \frac{\lambda^2 n^2}{m^2}\right)^2}{\frac{\lambda^2}{m^2}} + \frac{\frac{Z_b{}^2}{\pi^4}\frac{\lambda^2}{m^2}}{\left(1 + \frac{\lambda^2 n^2}{m^2}\right)^2} + \frac{\left(1 + \frac{\lambda^2 n^2}{m^2}\right)^2}{\frac{\lambda^2}{m^2}\left[1 + \psi_b\left(1 + \frac{\lambda^2 n^2}{m^2}\right)\right]} \tag{11}$$

where

$$\left.\begin{aligned}
k_b &= \frac{N^* b^2\left(B_1 + B_2\right)}{\pi^2 h^2 B_1 B_2} \\
Z_b{}^2 &= \frac{b^4\left(B_1 + B_2\right)^2\left(1 - \mu^2\right)}{r^2 h^2 B_1 B_2} \\
\psi_b &= \frac{\pi^2 d}{b^2} = \frac{\pi^2 c B_1 B_2}{b^2 G_c\left(B_1 + B_2\right)} \\
S &= \frac{D_1 + D_2}{\overline{d}} = \frac{\left(B_1 + B_2\right)\left(D_1 + D_2\right)}{h^2 B_1 B_2} \\
\lambda &= \frac{a}{b}
\end{aligned}\right\} \tag{12}$$

If the face-sheet flexural stiffnesses D_1 and D_2 are neglected as in reference 1, equation (11) with $S = 0$ becomes

$$k_b = \frac{Z_b{}^2}{\pi^4}\frac{\frac{\lambda^2}{m^2}}{\left(1 + \frac{\lambda^2 n^2}{m^2}\right)^2} + \frac{\left(1 + \frac{\lambda^2 n^2}{m^2}\right)^2}{\frac{\lambda^2}{m^2}\left[1 + \psi_b\left(1 + \frac{\lambda^2 n^2}{m^2}\right)\right]} \tag{13}$$

Equation (13) is identical with an equation developed in reference 1 except that the core-stiffness and curvature parameters used herein are more general and take into account the asymmetry of the sandwich section.

It should be noted that equation (11) is quite general for all ranges of the parameters and is applicable for both curved plates and cylinders. If the core thickness is zero $(c = 0)$, equation (11) reduces to a Donnell-type (ref. 5) result for a bilayered shell. With $c = 0$ and the face sheets symmetrical, it reduces to the equation given in reference 6 for an isotropic shell.

The buckling load for a shell of finite length can be obtained by minimizing equation (11) with respect to m and n. For an infinitely long shell the minimization should be carried out with respect to n and to the buckle-wavelength ratio $\frac{m}{\lambda n}$.

Plate of Infinite Length

Equation (11) has been minimized with respect to n and $\frac{m}{\lambda n}$ and has been applied to an infinitely long curved plate. The results are given by the solid lines in figure 3. It should be noted that the results for $S = 0$ shown in figure 3 agree with those in reference 1 and that the results in figure 3(a) for $S = 1/3$ and $c = 0$ $\left(t_1 = t_2 = h\right)$ agree with those for an isotropic homogeneous curved plate given in reference 6.

Stein and Mayers (ref. 1) give the following formulas for the critical load of an infinitely long curved plate obtained from an equation of the form of equation (13):

(1) For $\frac{Z_b}{\pi^2} < \frac{4\sqrt{1 - \psi_b}}{\left(1 + \psi_b\right)^2}$,

$$k_b \approx \frac{4}{\left(1 + \psi_b\right)^2} + \frac{Z_b^2}{\pi^4} \frac{1 - \psi_b}{4} \tag{14}$$

(2) For $\frac{4\sqrt{1 - \psi_b}}{\left(1 + \psi_b\right)^2} < \frac{Z_b}{\pi^2} \leqq \frac{\sqrt{1 - \psi_b}}{\psi_b}$,

$$k_b \approx \frac{\frac{Z_b}{\pi^2}}{\sqrt{1 - \psi_b}}\left(2 - \frac{\frac{Z_b}{\pi^2} \psi_b}{\sqrt{1 - \psi_b}}\right) \tag{15}$$

(3) For $\frac{Z_b}{\pi^2} \geqq \frac{\sqrt{1 - \psi_b}}{\psi_b}$,

$$k_b = \frac{1}{\psi_b} \tag{16}$$

Although figure 3 shows plots of the critical load for all ranges of the parameters, it is of interest to develop simple formulas which apply for the more important ranges. In particular, the range where S is less than 0.01 is of practical importance. For a symmetrical sandwich this range corresponds to the case where t/h is less than about 1/6. As is seen from figure 3, it is reasonable to consider equations (14) and (15) to be applicable to sandwiches where the face-sheet thickness is small with respect to the depth of the sandwich. On the other hand, equation (16) is not adequate even for the range $S < 0.01$; that is, the face-sheet-flexural-stiffness contribution is quite significant even when the thickness is small.

It is well known that equation (16) is used for $S = 0$ in the parameter range of

$$\frac{Z_b}{\pi^2} \geqq \frac{\sqrt{1 - \psi_b}}{\psi_b}$$

and corresponds to large core shearing deformations and thereby to an infinite wavelength ratio $\frac{m}{\lambda n}$. In order to develop an equation to replace equation (16) for a small value of $S > 0$, the contribution of the face sheets to the critical load of the plate in the range of large $\frac{m}{\lambda n}$ is investigated.

In the study of this contribution for the infinitely long plate, n is set equal to 1 in equation (11) and the equation is rewritten in the following form:

$$k_b = \frac{S\left(1 + \frac{m^2}{\lambda^2}\right)^2}{\frac{m^2}{\lambda^2}} + \frac{\frac{{Z_b}^2}{\pi^4}\frac{m^2}{\lambda^2}}{\left(1 + \frac{m^2}{\lambda^2}\right)^2} + \frac{\left(1 + \frac{m^2}{\lambda^2}\right)^2}{\frac{m^2}{\lambda^2}\left[1 + \psi_b\left(1 + \frac{m^2}{\lambda^2}\right)\right]} \tag{17}$$

In a similar fashion, equation (13) becomes

$$k_b = \frac{\frac{Z_b^2}{\pi^4}\frac{m^2}{\lambda^2}}{\left(1 + \frac{m^2}{\lambda^2}\right)^2} + \frac{\left(1 + \frac{m^2}{\lambda^2}\right)^2}{\frac{m^2}{\lambda^2}\left[1 + \psi_b\left(1 + \frac{m^2}{\lambda^2}\right)\right]} \tag{18}$$

It is clear from equation (18) that as the wavelength ratio $\frac{m}{\lambda} = \frac{b}{a/m}$ approaches infinity, equation (18) becomes equation (16). In equation (17), however, as $\frac{m}{\lambda}$ becomes large the contribution of the first term and consequently the buckling load become very large. This inconsistency for large values of $\frac{m}{\lambda}$ between equations (17) and (18) reflects the effect of the flexural stiffness of the face sheets on the critical load.

In order to determine the proper contribution of the face sheets to the buckling of the shell, equation (17) should be minimized with respect to $\frac{m}{\lambda}$. Unfortunately, such a procedure does not yield any simple results. Attention is therefore focused on the range of parameters where both $\left(\frac{m}{\lambda}\right)^2 \gg \frac{1}{\psi_b}$ and $\left(\frac{m}{\lambda}\right)^2 \gg 1$ so that equation (17) can be written as

$$k_b \approx S\left(\frac{m}{\lambda}\right)^2 + \frac{\frac{Z_b^2}{\pi^4}}{\left(\frac{m}{\lambda}\right)^2} + \frac{1}{\psi_b}$$

Minimizing k_b with respect to $\frac{m}{\lambda}$ leads to

$$\frac{\partial k_b}{\partial\left(\frac{m}{\lambda}\right)} \approx 2S\left(\frac{m}{\lambda}\right) - 2\,\frac{\frac{Z_b^2}{\pi^4}}{\left(\frac{m}{\lambda}\right)^3} = 0$$

or

$$\left(\frac{m}{\lambda}\right)^4 = \frac{Z_b^2}{\pi^4 S}$$

The critical buckling-load coefficient becomes therefore

$$k_b \approx \frac{1}{\psi_b} + \frac{2Z_b\sqrt{S}}{\pi^2} \tag{19}$$

For a symmetrical sandwich the critical force is

$$N_{cr} = \frac{G_c h^2}{c}\left[1 + \frac{2}{\sqrt{3\left(1 - \mu^2\right)}} \frac{c}{r} \frac{t^2}{h^2} \frac{E}{G_c}\right] \tag{20}$$

The usual result, which is obtained by neglecting the faces, is

$$N_{cr} = \frac{G_c h^2}{c}$$

The results from equation (19) are shown in figure 3 as dashed lines when they differ from the exact results obtained by minimizing equation (11). The plots indicate that equation (19) is quite accurate for small values of S in the range of interest, namely

$$\frac{Z_b}{\pi^2} \geqq \frac{\sqrt{1 - \psi_b}}{\psi_b}$$

Equation (19) is also seen to apply for large values of S if Z_b is also large and is in fact the asymptote for the results obtained by an exact minimization procedure.

It is interesting to note that for a symmetrical sandwich the critical force N_{cr} given by equation (20) is made up of the sum of three classical buckling forces for three different elements. These forces are -

The classical sandwich force obtained by neglecting the face-sheet flexural rigidity $\left(\frac{G_c h^2}{c}\right)$; and

The classical buckling force for each of the two face sheets considered separately $\left(\frac{1}{\sqrt{3\left(1 - \mu^2\right)}} \frac{Et^2}{r}\right)$.

When the face sheets are not alike, the result is somewhat more complicated.

Equation (11) can also be used to determine the critical load of a closed, simply supported cylinder of arbitrary length. For a cylinder, however, $b = 2\pi r$ and n is restricted to even integers. Plots similar to those in figure 3 could be prepared for the infinitely long cylinder; however, this work was not undertaken in the present study since the trends would be similar to those for the curved plate. The major point of practical interest is the development of a simple expression for cylinders which indicates the contribution of the face sheets in the range of large values of Z_a. In order to investigate the cylinder it is convenient to rewrite equation (11) as

$$k_a = Sm^2\left(1 + \frac{\lambda^2 n^2}{m^2}\right)^2 + \frac{\dfrac{{Z_a}^2}{\pi^4}}{m^2\left(1 + \dfrac{\lambda^2 n^2}{m^2}\right)^2} + \frac{m^2\left(1 + \dfrac{\lambda^2 n^2}{m^2}\right)^2}{1 + \psi_a m^2\left(1 + \dfrac{\lambda^2 n^2}{m^2}\right)} \tag{21}$$

or, if the face-sheet stiffness is neglected ($S = 0$),

$$k_a = \frac{\dfrac{{Z_a}^2}{\pi^4}}{m^2\left(1 + \dfrac{\lambda^2 n^2}{m^2}\right)^2} + \frac{m^2\left(1 + \dfrac{\lambda^2 n^2}{m^2}\right)^2}{1 + \psi_a m^2\left(1 + \dfrac{\lambda^2 n^2}{m^2}\right)} \tag{22}$$

where

$$\left.\begin{aligned}
k_a &= \frac{N^* a^2\left(B_1 + B_2\right)}{\pi^2 h^2 B_1 B_2} \\
{Z_a}^2 &= \frac{a^4\left(B_1 + B_2\right)^2\left(1 - \mu^2\right)}{r^2 h^2 B_1 B_2} \\
\psi_a &= \frac{\pi^2 c B_1 B_2}{a^2 G_c\left(B_1 + B_2\right)}
\end{aligned}\right\} \tag{23}$$

and where S and λ are the same as defined in equation (12).

Equation (22) is identical with an equation originally developed in reference 1, except that the core-stiffness and curvature parameters again take into account the asymmetry of the sandwich section. Likewise, in reference 1 the following formulas are given for the critical load of a simply supported cylinder obtained from an equation of the form of equation (22):

(1) For $\frac{Z_a}{\pi^2} \leqq \frac{1}{1 + \psi_a}$,

$$k_a = \frac{1}{1 + \psi_a} + \frac{{Z_a}^2}{\pi^4} \tag{24}$$

(2) For $\frac{1}{1 + \psi_a} \leqq \frac{Z_a}{\pi^2} \leqq \frac{1}{\psi_a}$,

$$k_a = \frac{Z_a}{\pi^2}\left(2 - \frac{Z_a}{\pi^2}\psi_a\right) \tag{25}$$

(3) For $\frac{Z_a}{\pi^2} \geqq \frac{1}{\psi_a}$,

$$k_a = \frac{1}{\psi_a} \tag{26}$$

In order to develop a simple expression which includes the contribution of the face sheets, it is useful to proceed as in the previous case for the curved plate and investigate the behavior of the cylinder in the parameter range $\frac{Z_a}{\pi^2} \geqq \frac{1}{\psi_a}$. This range corresponds to large shear deformations in the core and a large wavelength ratio $\frac{m}{\lambda n}$. When $\left(\frac{m}{\lambda n}\right)^2 >> \frac{1}{\psi_a}$ and $\left(\frac{m}{\lambda n}\right)^2 >> 1$, equation (21) becomes

$$k_a = Sm^2 + \frac{{Z_a}^2}{\pi^4 m^2} + \frac{1}{\psi_a}$$

Minimizing this equation with respect to m^2 results in the following expression for the critical load in the region where face-sheet flexural stiffness is significant $\left(\frac{Z_a}{\pi^2} \geqq \frac{1}{\psi_a}\right)$:

$$k_a \approx \frac{1}{\psi_a} + \frac{2Z_a\sqrt{S}}{\pi^2} \tag{27}$$

As might be expected, this formula is similar to equation (19), which was obtained for the curved plate.

CONCLUDING REMARKS

A study has been made of the effect of flexural stiffness of the face sheets on the buckling of elastic curved plates and cylindrical shells of sandwich construction subjected to axial compression. The results of the study for curved plates are given in graphical form. The results agree at one extreme with classical sandwich theory and at the other extreme with isotropic shell theory. Any face-sheet flexural stiffness is seen to increase significantly the buckling strength of these sandwich shells in certain ranges of the shell parameters where the core is very weak in shear. In order to obtain any meaningful results in these ranges, the effect of face-sheet flexural stiffness must be included. Simple formulas have been developed in the paper for the buckling load of curved plates and cylinders which take into account this face-sheet contribution in the range of parameters of practical interest where the face-sheet thickness is small.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., January 28, 1965.

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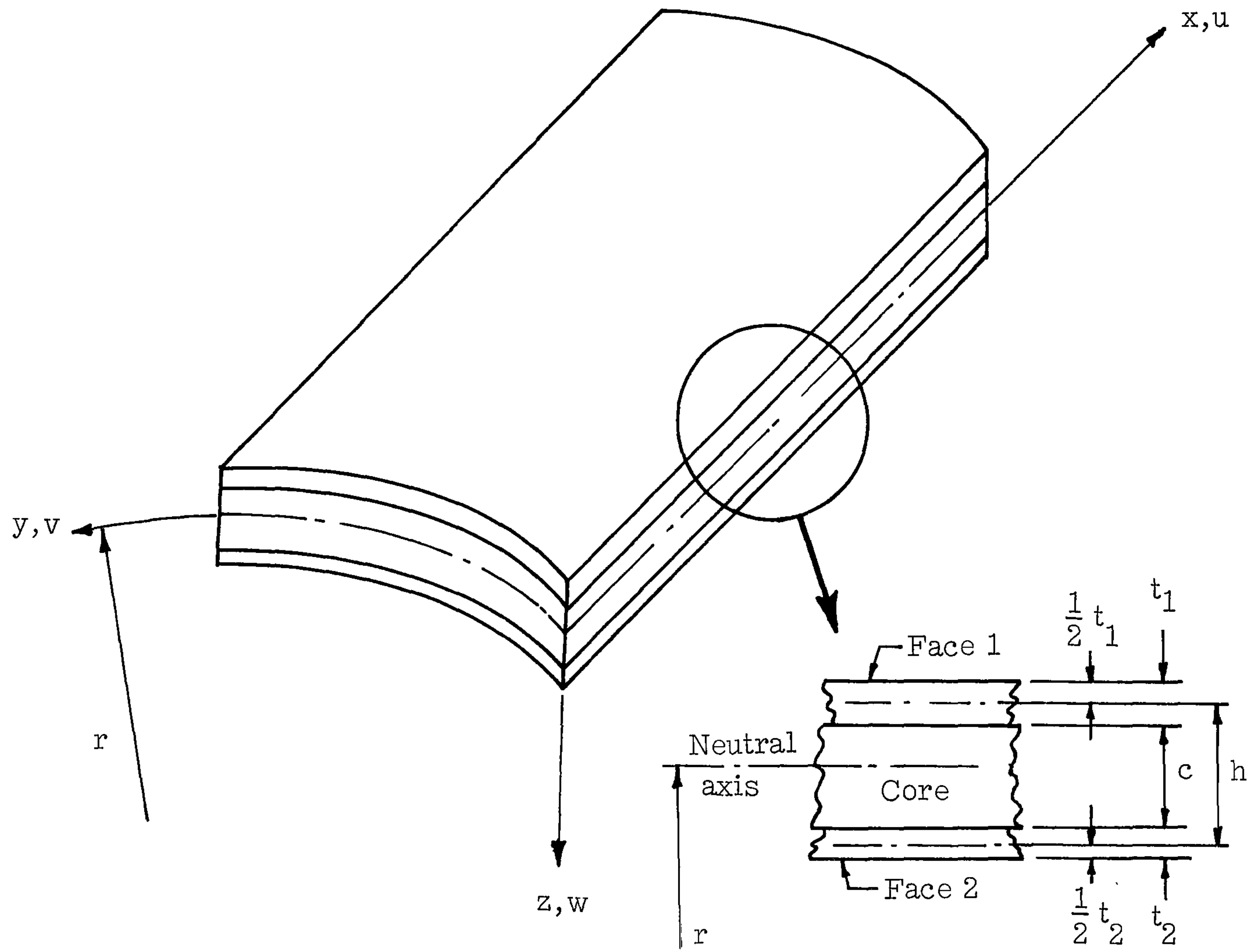


Figure 1.- Cylindrical-sandwich-shell element.

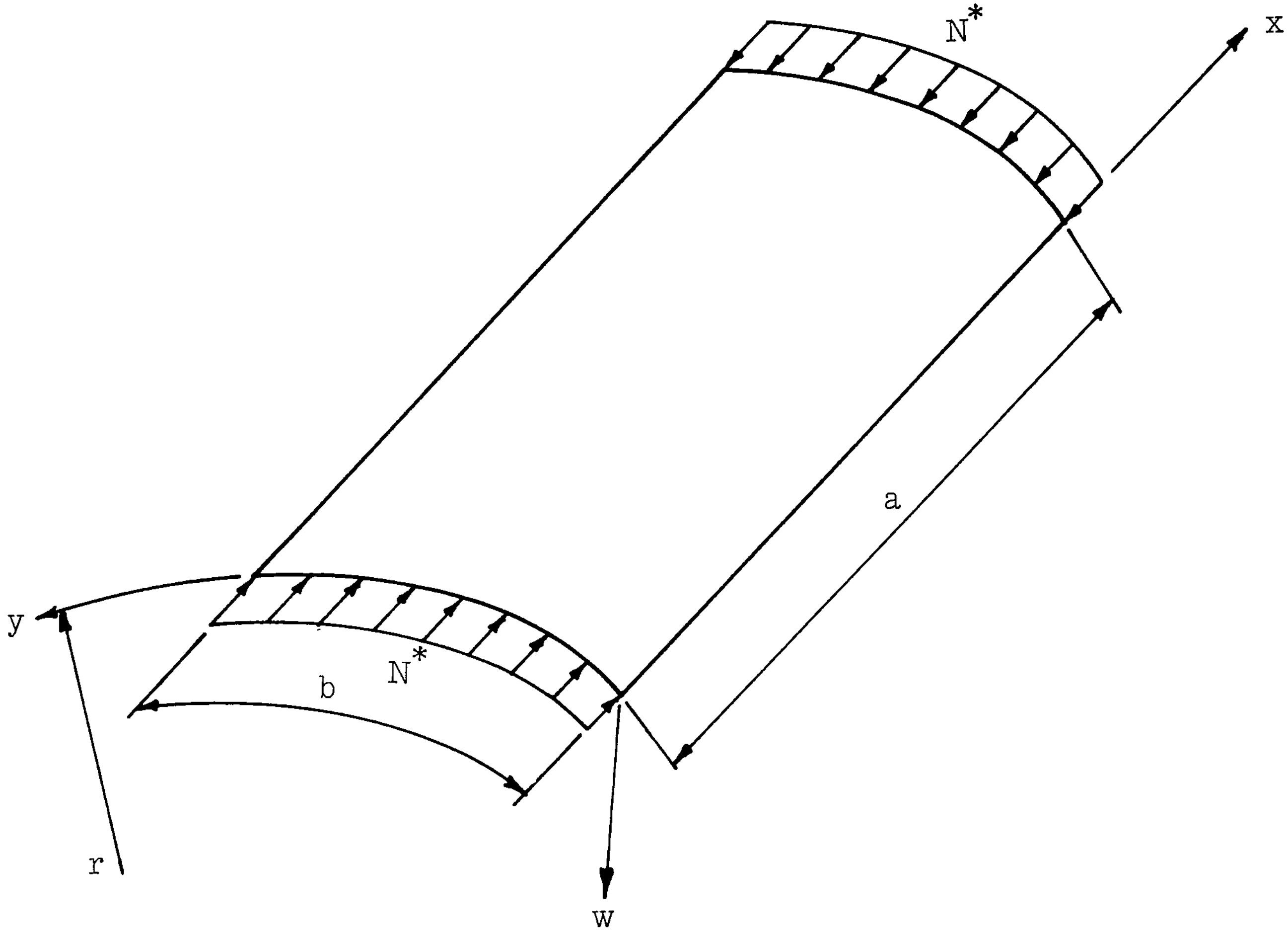
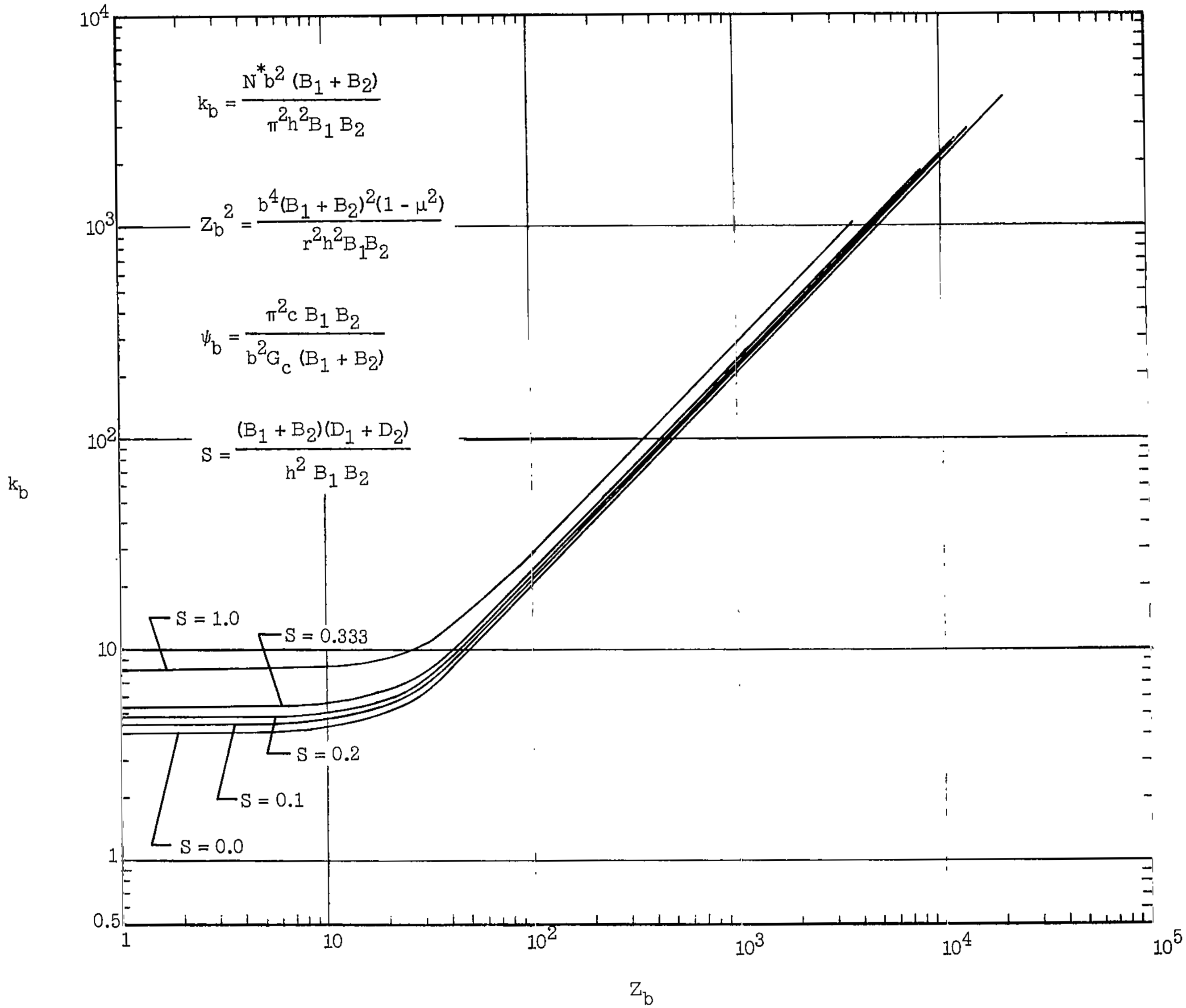
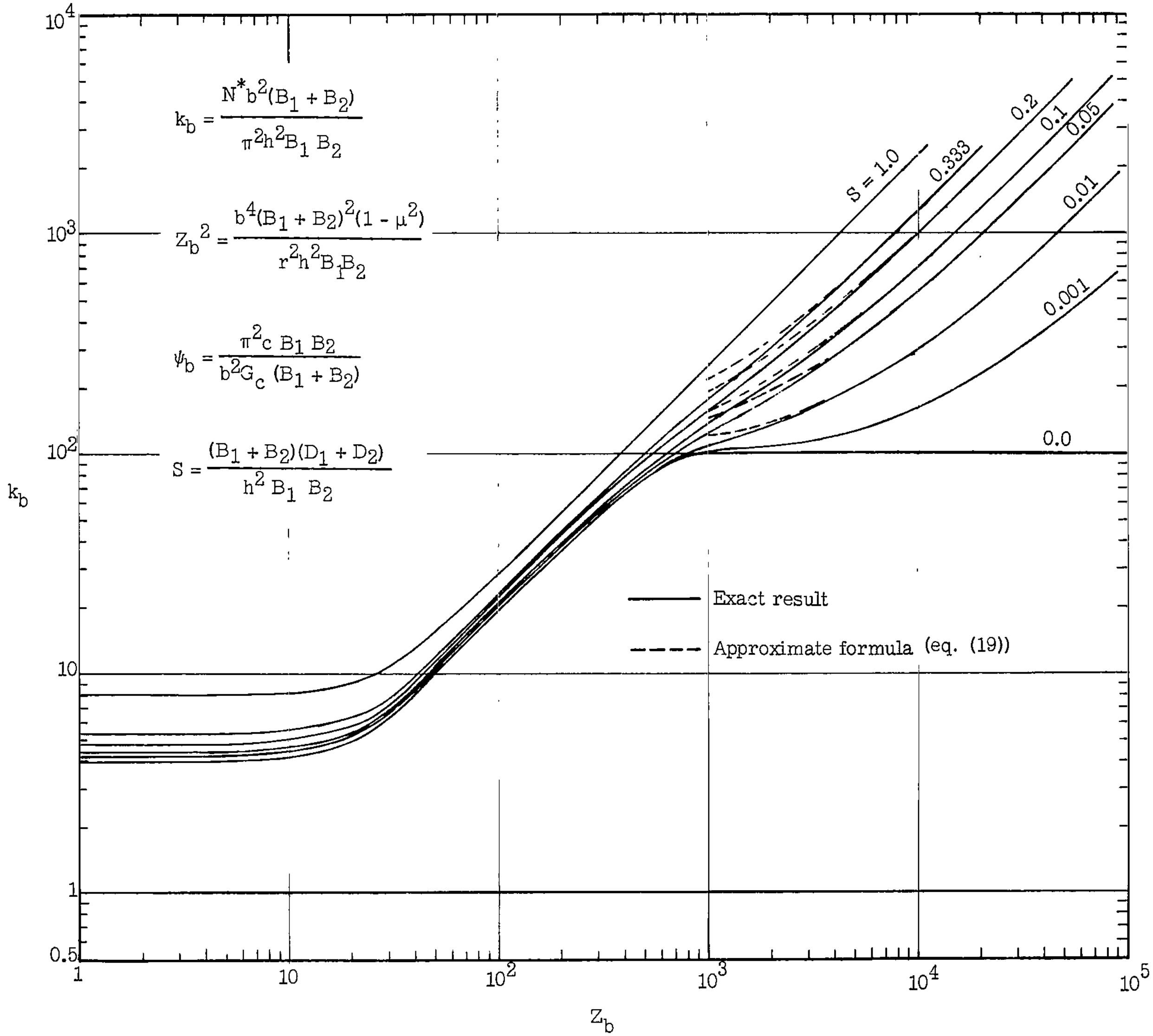


Figure 2.- Curved sandwich plate subjected to axial compression.
All edges are simply supported.



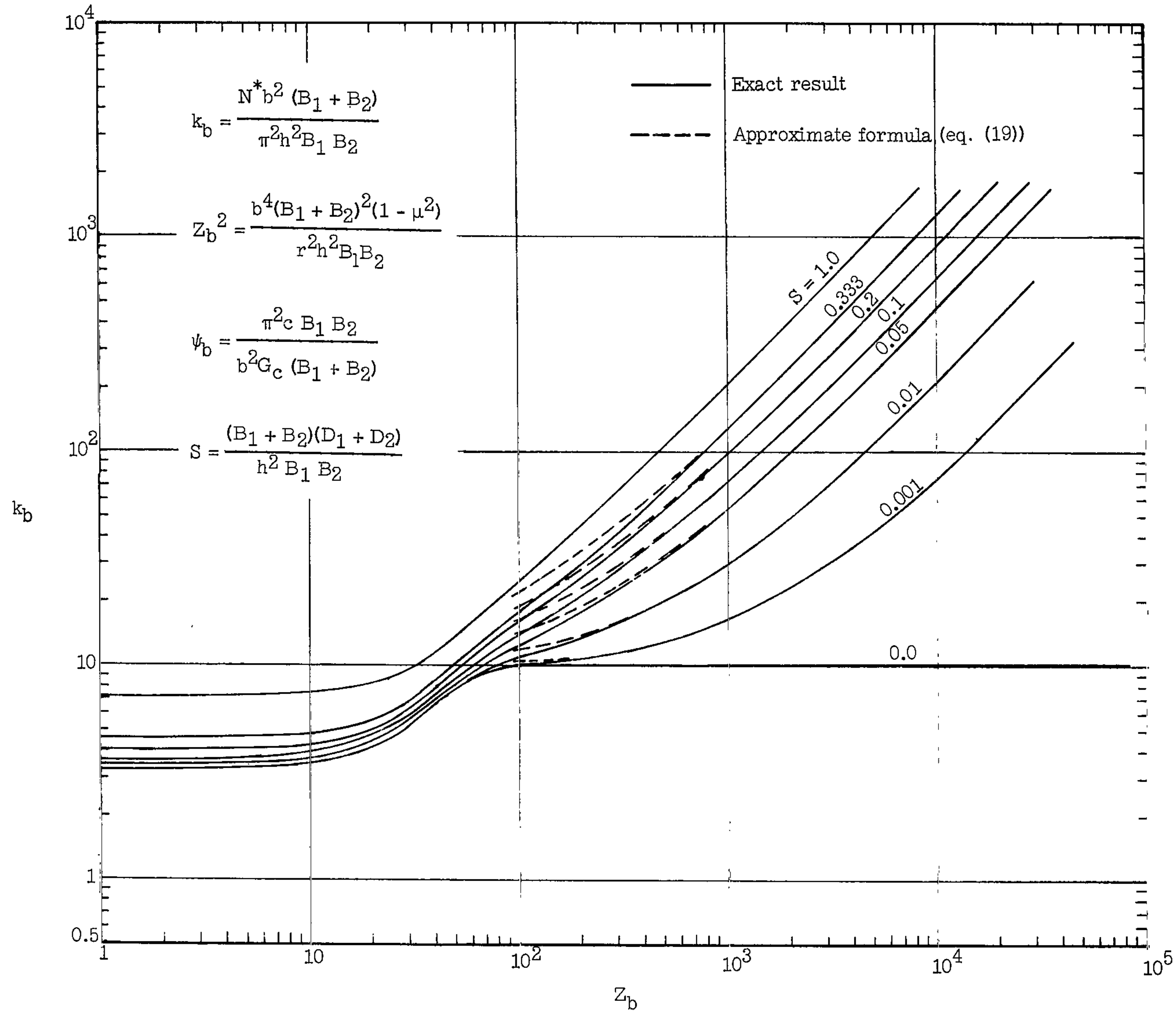
(a) $\psi_b = 0$.

Figure 3.- Buckling coefficients for simply supported, infinitely long, curved sandwich plates.



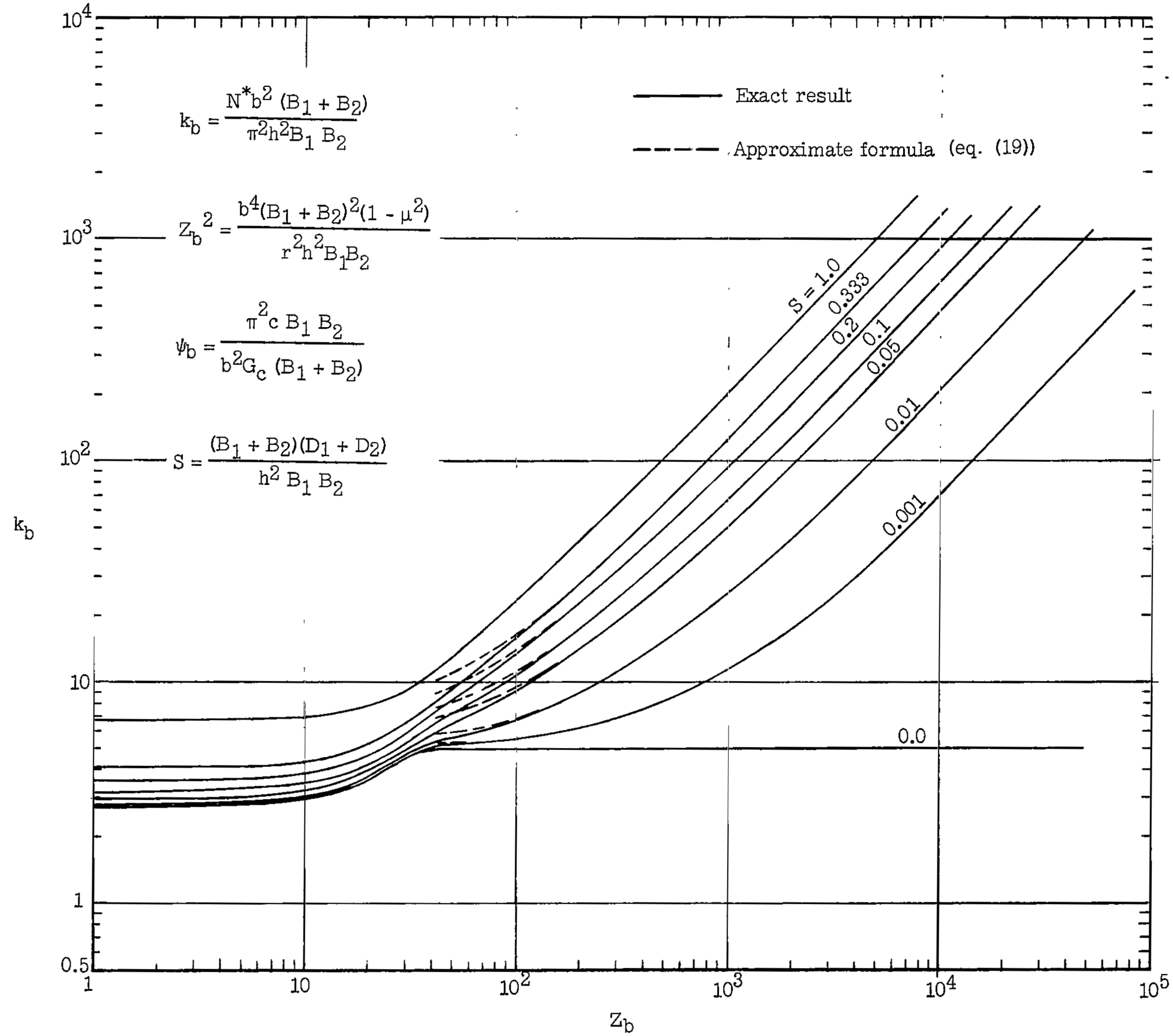
(b) $\psi_b = 0.01$.

Figure 3.- Continued.



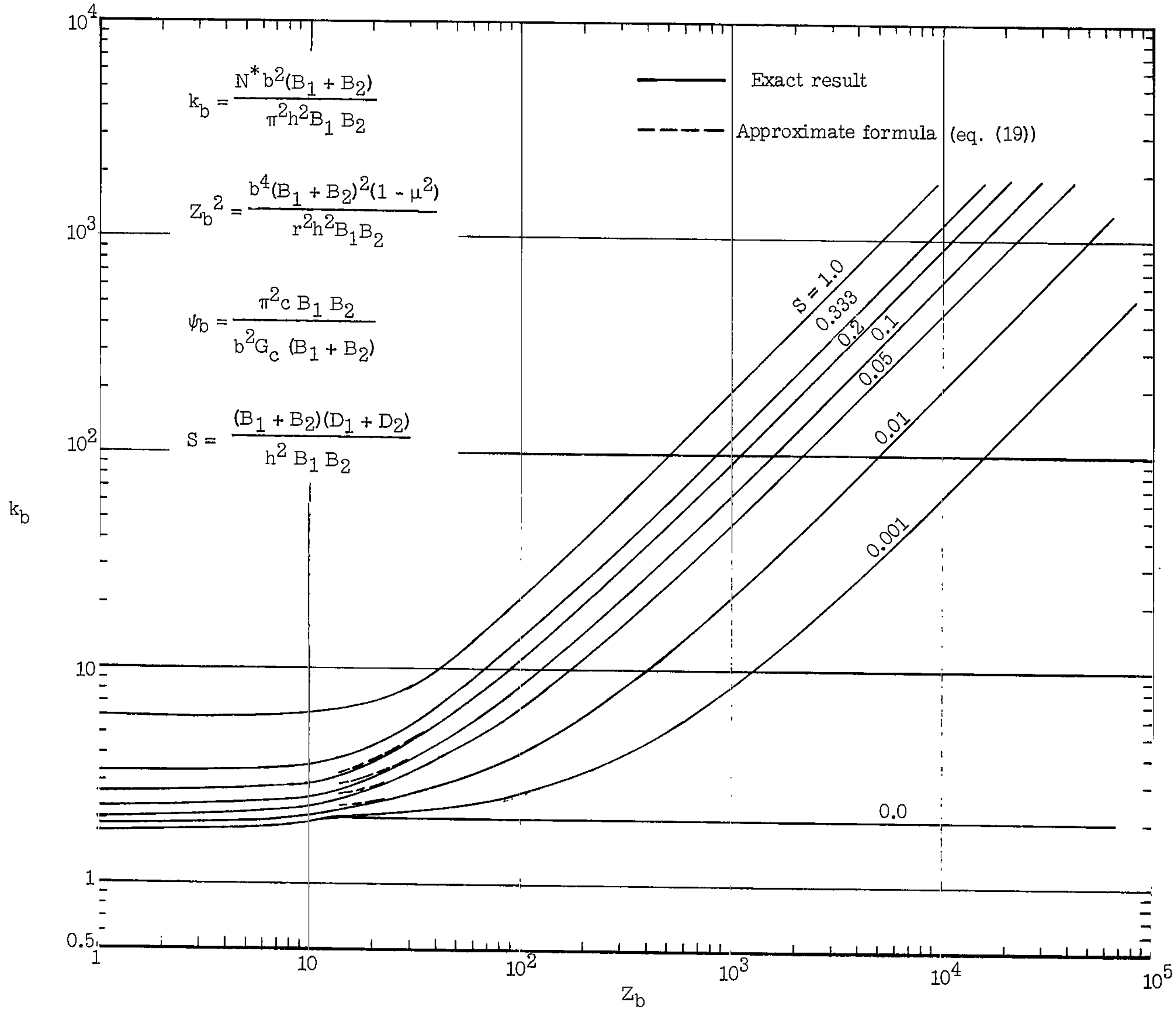
(c) $\psi_b = 0.1$.

Figure 3.- Continued.



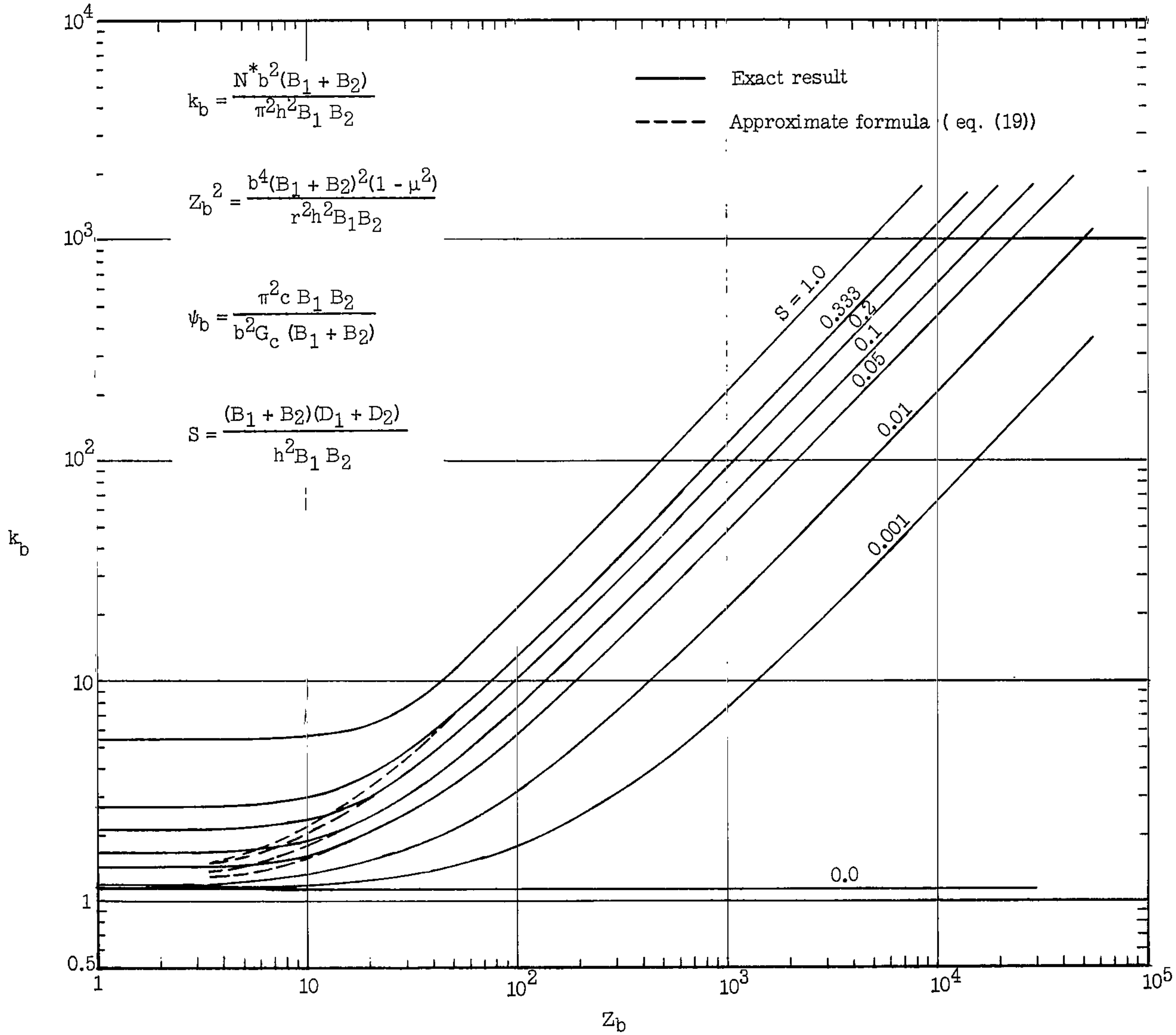
(d) $\psi_b = 0.2$.

Figure 3.- Continued.



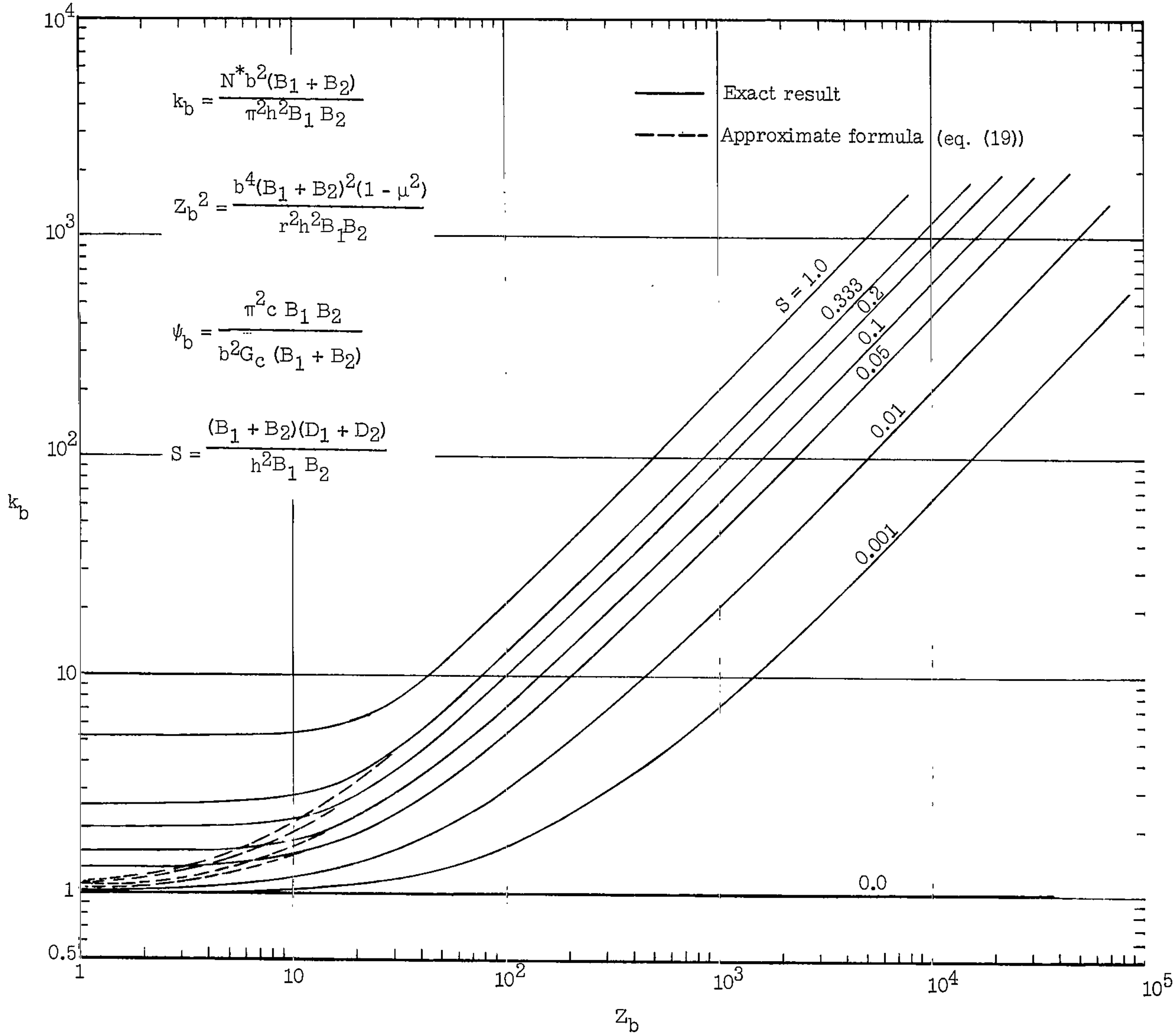
(e) $\psi_b = 0.5$.

Figure 3.- Continued.



(f) $\psi_b = 0.9$.

Figure 3.- Continued.



(g) $\psi_b = 1.0$.

Figure 3.- Concluded.

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